A new laminate theory:
simplified design, testing and manufacturing

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June 18, 2025

Difference between Quad and emerging DD

Quad

[ply angle]	0	45	-45	90
[ply#]	5	2	2	1

Fixed: ply angles

Variable: discrete ply #

Multiple families: 6-, 8-, 10-ply, …

No bounds, stacking sequence, D_{61}

DD

[ply angle]	75	15	-75	-15
[ply#]	1	1	1	1

Variable: Continuous angles

Fixed: ply #

One family, has practical limits: 4-ply; [D*] = [A*] when homogenized

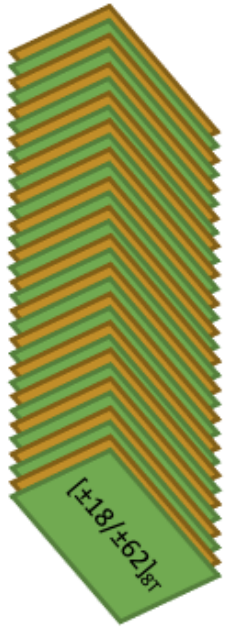
Relartive stacking of Quad vs DD

Quad

Quad lam: # plies	π/4: 8	Hard1: 12	Hard2: 16	Soft1: 16	Soft2: 20	Soft3: 20	Soft4: 32
Quad lam code	$[0/\pm45/90]_S$	$[0_3/\pm45/90]_S$	$[0_5/\pm45/90]_S$	$[0/\pm45_3/90]_S$	$[0_2/\pm45_3/90_2]_S$	$[0_3/\pm45_3/90]_S$	$[0_5/\pm45_4/90_3]_S$
Stacking permutations	24	600	10,488	10,488	166,080	166,080	>1,000,000

Matching DD

$[\pm22.5/\pm67.5]_{4T}$	$[\pm0/\pm52]_{6T}$	$[\pm0/\pm37]_{8T}$	$[\pm30/\pm60]_{8T}$	$[\pm25/\pm64]_{10T}$	$[\pm18/\pm57]_{10T}$	$[\pm16/\pm62]_{16T}$



DD

DD lam: # plies	Soft1: 4	Hard1: 4	Hard2: 4	Soft2: 4	Soft3: 4	Soft4: 4	Soft5: 4
Stacking permutations	2	2	2	2	2	2	2

Quad

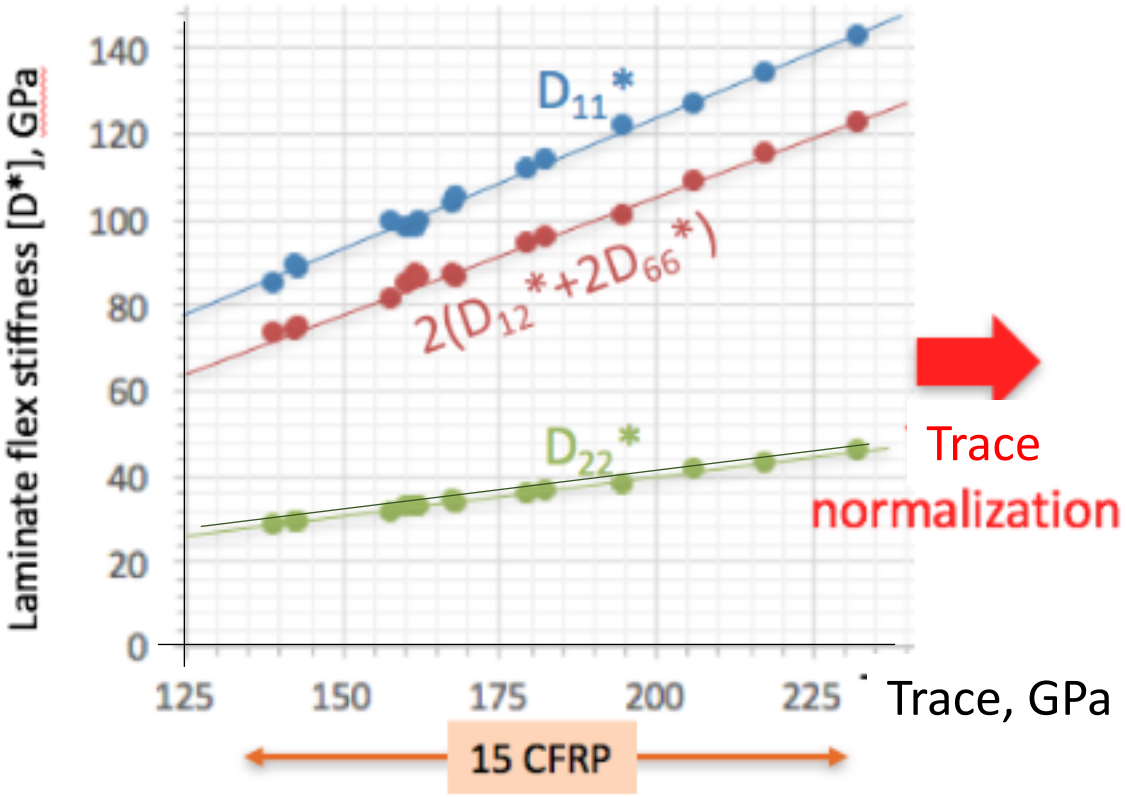
$$L = D_{11}\frac{\partial^4}{\partial x^4} + 4D_{16}\frac{\partial^4}{\partial x^3\,\partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial x^2\,\partial y^2} + 4D_{26}\frac{\partial^4}{\partial x\,\partial y^3} + D_{22}\frac{\partial^4}{\partial y^4}$$

DD

$$L = D_{11}\frac{\partial^4}{\partial x^4} + \qquad + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial x^2\,\partial y^2} + \qquad + D_{22}\frac{\partial^4}{\partial y^4}$$

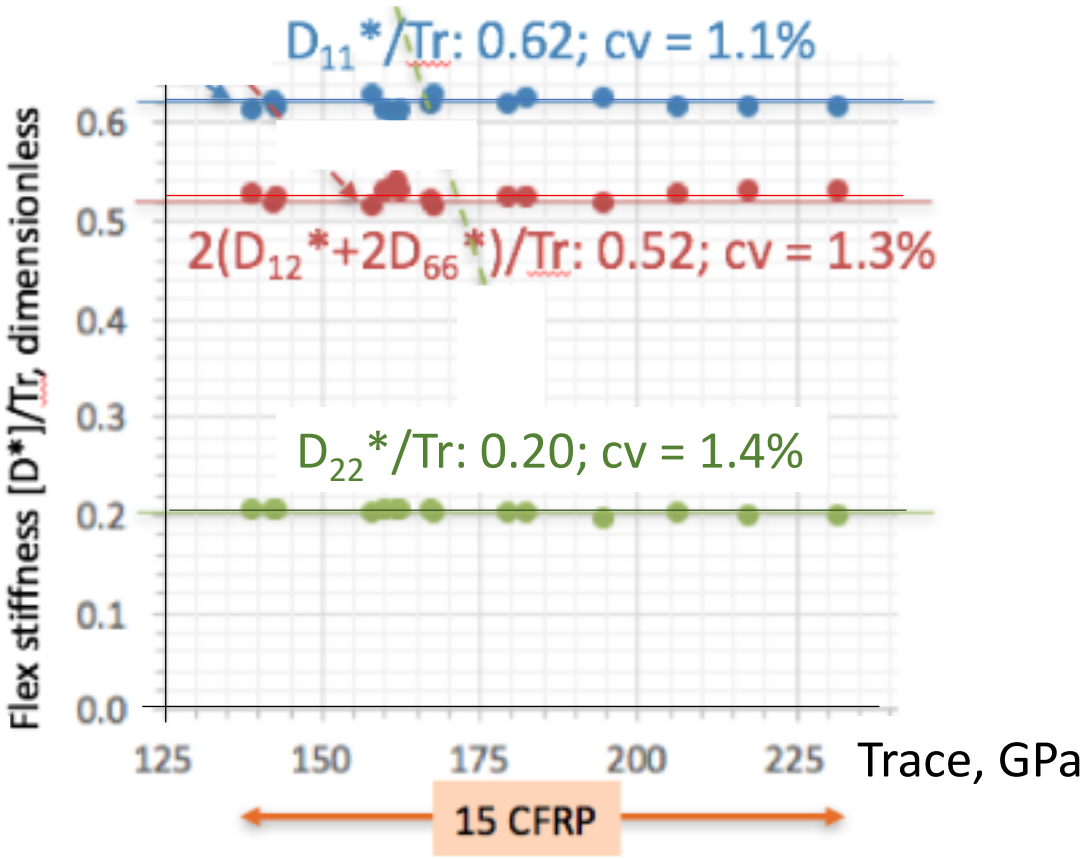
Hard laminate: $[\pm 0/\pm 50]_{6T}$

No master ply in flex for quad



Trace normalization

Material independent



Master elastic constants of CFRP laminates

Material

$Q_{xx} + Q_{yy} + 2*Q_{ss}$

Ply material	Trace
IM6/epoxy	232
IM7/977-3	218
T300/5208	206
IM7/MTM45	195
T800/Cytec	183
IM7/8552	180
T800S/3900	168
T300/F934	168
T700 C-Ply 64	163
AS4/H3501	162
T650/epoxy	160
T4708/MR60H	158
T700/2510	144
AS4/MTM45	143
T700 C-Ply 55	139

Master stiffness: material independent

A_{11}*, %

[±Φ] \ [±Ψ]	0	15	30	45	60	75	90	[±Ψ]
0	89							90
15	83	78						75
30	71	66	53					60
45	58	53	40	27				45
60	50	45	37	32	19	11		30
75	47	42	30	17	9	6		15
90	47	42	29	16	8	6	5	0
[±Φ]	90	75	60	45	30	15	0	A_{22}* [±Φ]

Square symmetric

[QI] at [22.5/67.5]

A_{66}*, %

[±Φ] \ [±Ψ]	0	15	30	45	60	75	90	[±Ψ]
0	3.1							90
15	5.5	8.0						75
30	10.4	12.9	17.7					60
45	12.9	15.3	20.2	22.6				45
60	10.4	12.9	13	17.7	20.2	17.7		30
75	5.5	8.0	12.9	15.3	12.9	8.0		15
90	3.1	5.5	10.4	12.9	10.4	5.5	3.1	0
[±Φ]	90	75	60	45	30	15	0	[±Φ]

Square symmetric

A12* = A66* - 1.5

E_1*, %

[±Φ] \ [±Ψ]	0	15	30	45	60	75	90	[±Ψ]
0	88							90
15	80	71						75
30	61	51	30					60
45	50	41	22	11				45
60	47	40	24	11	6			30
75	47	41	34	13	7	5.4		15
90	47	41	27	14	7	5.4	5.2	0
[±Φ]	90	75	60	45	30	15.0	0.0	E_2* [±Φ]

Square symmetric

ν_{21}*

[±Φ] \ [±Ψ]	0	15	30	45	60	75	90	[±Ψ]
0	0.32							90
15	0.74	1.10						75
30	1.08	1.32	1.43					60
45	0.70	0.83	0.97	0.77				45
60	0.31	0.39	0.51	0.47	0.31			30
75	0.10	0.10	0.26	0.26	0.17	0.08		15
90	0.04	0.09	0.18	0.20	0.13	0.05	0.02	0
[±Φ]	90	75	60	45	30	15	0	ν_{12}*

0.3

Omni FPF

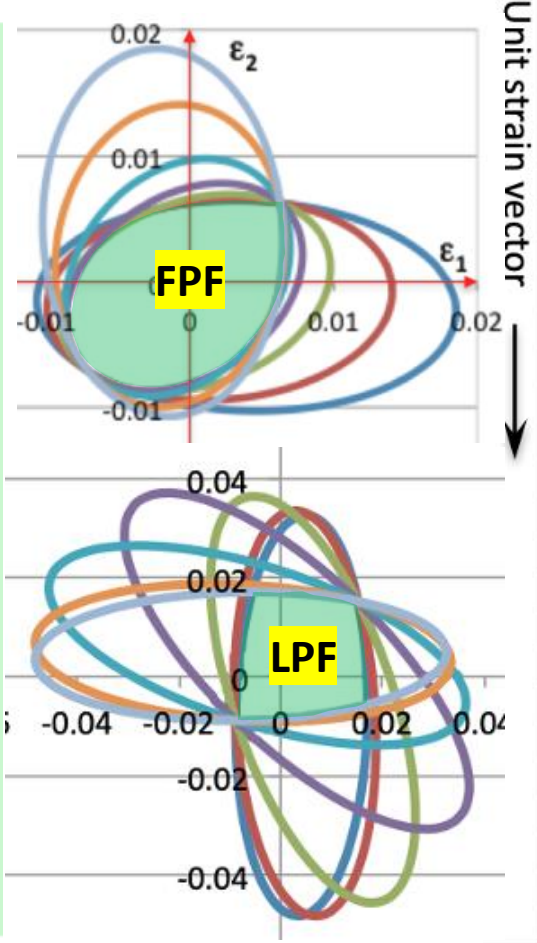
Control: all ply angles

Ply orientation →

Unit strain vector ↓

Angle	0	±15	±30	±45	±60	±75	90	FPF Omni
0	18.0	14.0	9.7	7.5	6.6	6.3	6.3	6.3
15	14.6	12.8	9.9	8.1	7.2	6.8	6.6	6.6
30	11.2	10.7	9.7	8.7	8.0	7.6	7.5	7.5
45	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9
60	7.5	7.6	8.0	8.7	9.7	10.7	11.2	7.5
75	6.6	6.8	7.2	8.1	9.9	12.8	14.6	6.6
90	6.3	6.3	6.6	7.5	9.7	14.0	18.0	6.3
105	6.2	6.2	6.3	7.1	9.2	13.8	18.5	6.2
120	6.4	6.3	6.3	6.8	8.6	12.6	16.3	6.3
135	7.0	6.7	6.4	6.8	8.3	11.3	13.7	6.4
150	8.0	7.4	6.8	7.0	8.1	10.3	11.8	6.8
165	9.3	8.4	7.5	7.4	8.2	9.7	10.5	7.4
180	10.4	9.5	8.4	8.1	8.5	9.4	9.8	8.1
195	10.9	10.3	9.3	8.9	9.0	9.3	9.5	8.9
210	10.5	10.3	10.0	9.7	9.6	9.6	9.6	9.6
225	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
240	9.6	9.6	9.6	9.7	10.0	10.3	10.5	9.6
255	9.5	9.3	9.0	8.9	9.3	10.3	10.9	8.9
270	9.8	9.4	8.5	8.1	8.4	9.5	10.4	8.1
285	10.5	9.7	8.2	7.4	7.5	8.4	9.3	7.4
300	11.8	10.3	8.1	7.0	6.8	7.4	8.0	6.8
315	13.7	11.3	8.3	6.8	6.4	6.7	7.0	6.4
330	16.3	12.6	8.6	6.8	6.3	6.3	6.4	6.3
345	18.5	13.8	9.2	7.1	6.3	6.2	6.2	6.2
360	18.0	14.0	9.7	7.5	6.6	6.3	6.3	6.3

IM7/977



Omni LPF

Control: [90], [0] only

Ply orientation →

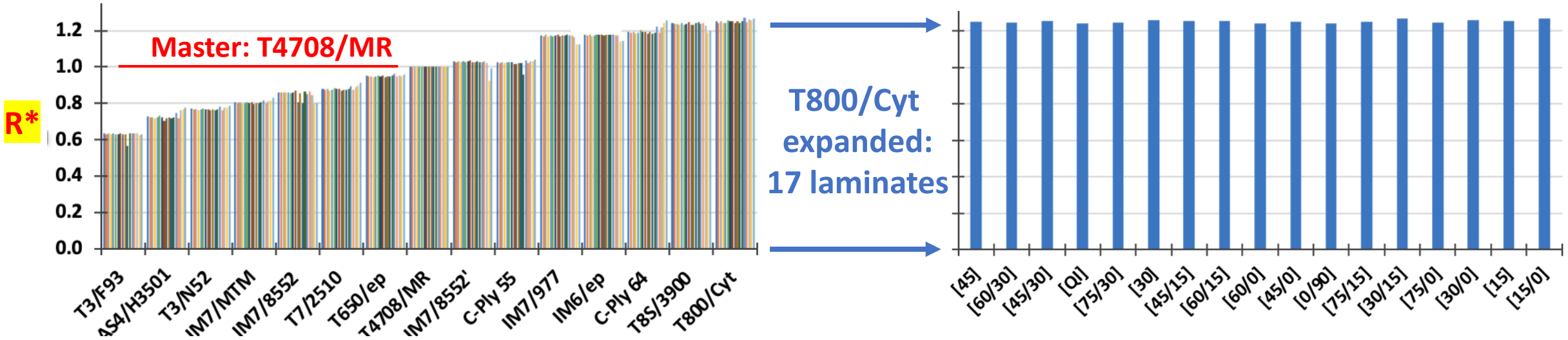
Unit strain vector ↓

Angle	0	±15	±30	±45	±60	±75	90	LPF Omni
0	17.0	18.0	21.4	27.9	34.7	32.6	29.6	17.0
15	17.3	18.0	20.2	24.1	28.6	31.5	32.2	17.3
30	18.7	19.1	20.3	22.1	24.1	25.7	26.3	18.7
45	21.5	21.5	21.5	21.5	21.5	21.5	21.5	21.5
60	26.3	25.7	24.1	22.1	20.3	19.1	18.7	18.7
75	32.2	31.5	28.6	24.1	20.2	18.0	17.3	17.3
90	29.6	32.6	34.7	27.9	21.4	18.0	17.0	17.0
105	19.6	23.3	36.0	34.4	24.1	19.1	17.7	17.7
120	13.6	15.9	27.1	42.8	29.0	21.6	19.7	13.6
135	10.7	12.2	19.1	43.2	37.4	26.3	23.5	10.7
150	9.2	10.2	14.7	31.2	47.9	34.8	30.6	9.2
165	8.5	9.3	12.4	21.8	44.6	46.6	42.4	8.5
180	8.4	9.0	11.3	16.8	28.9	43.2	46.8	8.4
195	8.9	9.4	11.0	14.2	19.5	25.8	28.8	8.9
210	10.1	10.4	11.3	12.9	14.8	16.7	17.4	10.1
225	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5
240	17.4	16.7	14.8	12.9	11.3	10.4	10.1	10.1
255	28.8	25.8	19.5	14.2	11.0	9.4	8.9	8.9
270	46.8	43.2	28.9	16.8	11.3	9.0	8.4	8.4
285	42.4	46.6	44.6	21.8	12.4	9.3	8.5	8.5
300	30.6	34.8	47.9	31.2	14.7	10.2	9.2	9.2
315	23.5	26.3	37.4	43.2	19.1	12.2	10.7	10.7
330	19.7	21.6	29.0	42.8	27.1	15.9	13.6	13.6
345	17.7	19.1	24.1	34.4	36.0	23.3	19.6	17.7
360	17.0	18.0	21.4	27.9	34.7	32.6	29.6	17.0

Material rating

Error prone

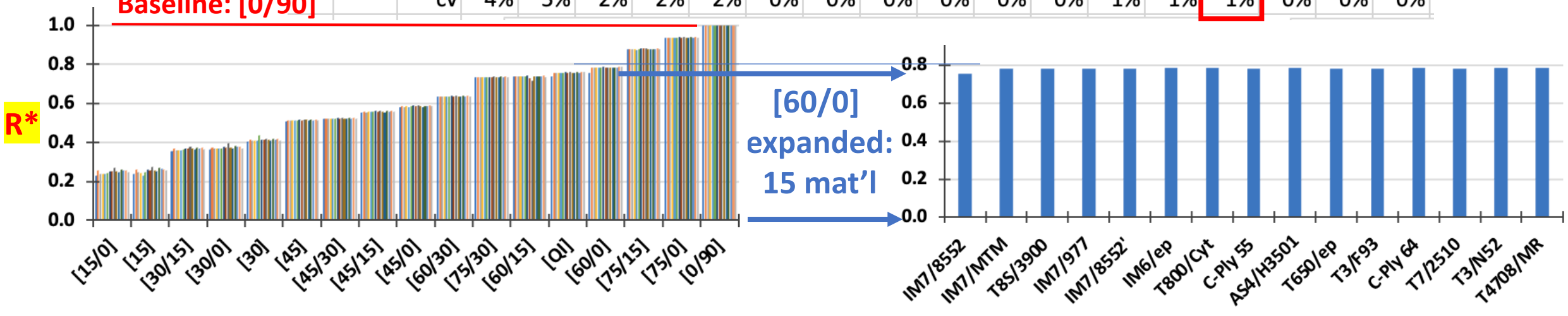
	Material rating	[45]	60/30	45/30	[QI]	75/30	[30]	45/15	60/15	[60/0]	[45/0]	[0/90]	75/15	30/15	[75/0]	[30/0]	[15]	[15/0]	Mat rati	R*	cv
11	T3/F93	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.64	0.63	0.63	0.57	0.64	0.64	0.63	0.63	0.62	0.63		0.63	2.6%
9	AS4/H3501	0.73	0.72	0.73	0.72	0.72	0.73	0.73	0.70	0.72	0.72	0.72	0.72	0.75	0.72	0.76	0.77	0.78		0.73	2.8%
14	T3/N52	0.77	0.77	0.77	0.76	0.77	0.77	0.77	0.77	0.76	0.77	0.76	0.77	0.78	0.76	0.78	0.78	0.79		0.77	1.0%
2	IM7/MTM	0.81	0.80	0.81	0.80	0.80	0.81	0.80	0.81	0.80	0.80	0.80	0.81	0.82	0.80	0.81	0.82	0.83		0.81	1.0%
1	IM7/8552	0.86	0.86	0.86	0.86	0.86	0.85	0.86	0.87	0.81	0.86	0.80	0.87	0.85	0.86	0.85	0.80	0.80		0.85	3.0%
13	T7/2510	0.88	0.88	0.88	0.87	0.88	0.88	0.88	0.88	0.87	0.88	0.88	0.88	0.90	0.88	0.89	0.90	0.91		0.88	1.3%
10	T650/ep	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.94	0.95	0.95	0.95	0.96	0.95	0.95	0.95	0.96		0.95	0.5%
15	T4708/MR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00	0.0%
5	IM7/8552'	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.02	1.03	1.03	1.03	1.03	1.03	1.02	0.92	0.99		1.02	2.6%
8	C-Ply 55	1.03	1.02	1.03	1.02	1.03	1.03	1.03	1.02	1.02	1.02	1.02	0.96	1.04	1.02	1.03	1.03	1.04		1.02	1.7%
4	IM7/977	1.18	1.17	1.18	1.17	1.17	1.17	1.17	1.18	1.17	1.17	1.17	1.18	1.17	1.17	1.16	1.12	1.13		1.17	1.4%
6	IM6/ep	1.18	1.18	1.18	1.17	1.17	1.18	1.18	1.18	1.17	1.18	1.18	1.18	1.18	1.17	1.18	1.14	1.15		1.17	1.0%
12	C-Ply 64	1.19	1.19	1.19	1.18	1.19	1.20	1.19	1.19	1.18	1.19	1.19	1.19	1.22	1.19	1.22	1.24	1.26		1.20	1.8%
3	T8S/3900	1.24	1.24	1.24	1.23	1.24	1.23	1.24	1.25	1.23	1.23	1.24	1.25	1.24	1.24	1.23	1.19	1.20		1.23	1.3%
7	T800/Cyt	1.25	1.24	1.25	1.24	1.24	1.26	1.25	1.25	1.24	1.25	1.24	1.25	1.27	1.25	1.26	1.26	1.27		1.25	0.7%



Laminate rating

Laminate rating		[15/0]	[15]	30/15	[30/0]	[30]	[45]	45/30	45/15	[45/0]	60/30	75/30	60/15	[QI]	[60/0]	75/15	[75/0]	[0/90]
1	IM7/8552	0.23	0.24	0.36	0.36	0.41	0.51	0.52	0.55	0.58	0.63	0.73	0.74	0.74	0.76	0.88	0.94	1.00
2	IM7/MTM	0.26	0.26	0.37	0.38	0.41	0.51	0.52	0.56	0.58	0.64	0.73	0.74	0.76	0.78	0.88	0.94	1.00
3	T8S/3900	0.24	0.25	0.36	0.37	0.41	0.51	0.52	0.55	0.58	0.63	0.73	0.74	0.76	0.78	0.88	0.94	1.00
4	IM7/977	0.24	0.25	0.36	0.37	0.41	0.51	0.52	0.56	0.58	0.63	0.73	0.74	0.76	0.78	0.88	0.94	1.00
5	IM7/8552'	0.24	0.23	0.36	0.37	0.41	0.51	0.52	0.56	0.58	0.63	0.73	0.74	0.76	0.78	0.87	0.94	1.00
6	IM6/ep	0.24	0.25	0.36	0.37	0.44	0.51	0.52	0.56	0.58	0.64	0.73	0.74	0.76	0.78	0.88	0.94	1.00
7	T800/Cyt	0.25	0.26	0.37	0.38	0.42	0.52	0.53	0.56	0.59	0.64	0.74	0.74	0.76	0.79	0.88	0.94	1.00
8	C-Ply 55	0.25	0.26	0.37	0.37	0.41	0.51	0.52	0.56	0.58	0.64	0.73	0.73	0.76	0.78	0.88	0.94	1.00
9	AS4/H3501	0.27	0.27	0.38	0.40	0.42	0.52	0.53	0.56	0.59	0.64	0.74	0.72	0.76	0.78	0.88	0.94	1.00
10	T650/ep	0.25	0.26	0.37	0.37	0.41	0.52	0.52	0.56	0.58	0.64	0.74	0.74	0.76	0.78	0.88	0.94	1.00
11	T3/F93	0.25	0.25	0.36	0.37	0.41	0.51	0.52	0.56	0.58	0.63	0.74	0.74	0.76	0.78	0.88	0.94	1.00
12	C-Ply 64	0.26	0.27	0.37	0.38	0.42	0.52	0.53	0.56	0.59	0.64	0.74	0.74	0.76	0.78	0.88	0.94	1.00
13	T7/2510	0.26	0.26	0.37	0.38	0.42	0.52	0.52	0.56	0.58	0.64	0.74	0.74	0.76	0.78	0.88	0.94	1.00
14	T3/N52	0.26	0.26	0.37	0.38	0.42	0.52	0.53	0.56	0.59	0.64	0.74	0.74	0.76	0.79	0.88	0.94	1.00
15	T4708/MR	0.25	0.26	0.36	0.37	0.41	0.51	0.52	0.56	0.58	0.64	0.73	0.73	0.76	0.79	0.88	0.94	1.00
	Laminate rating	0.25	0.25	0.37	0.37	0.41	0.51	0.52	0.56	0.58	0.64	0.73	0.74	0.76	0.78	0.88	0.94	1.00
	cv	4%	5%	2%	2%	2%	0%	0%	0%	0%	0%	0%	1%	1%	1%	0%	0%	0%

R*



Property ratings

Master DD plots for CFRP

	Material	Stiffness Trace	Strength R	Strength rank
5	T800/Cytec	183	1425	1.26
7	T800S/3900	168	1391	1.23
9	T700 C-Ply 64	163	1391	1.23
1	IM6/epoxy	232	1335	1.18
2	IM7/977-3	218	1323	1.17
14	T700 C-Ply 55	139	1176	1.04
4	IM7/MTM45	195	1154	1.02
11	T4708/MR60	158	1131	1.00
13	T650/epoxy	160	1086	0.96
12	T700/2510	144	1007	0.89
6	IM7/8552	180	950	0.84
3	T300/5208	206	882	0.78
10	AS4/H3501	162	848	0.75
8	T300/F934	168	713	0.63

Master R°

T800/Cyt

Master R

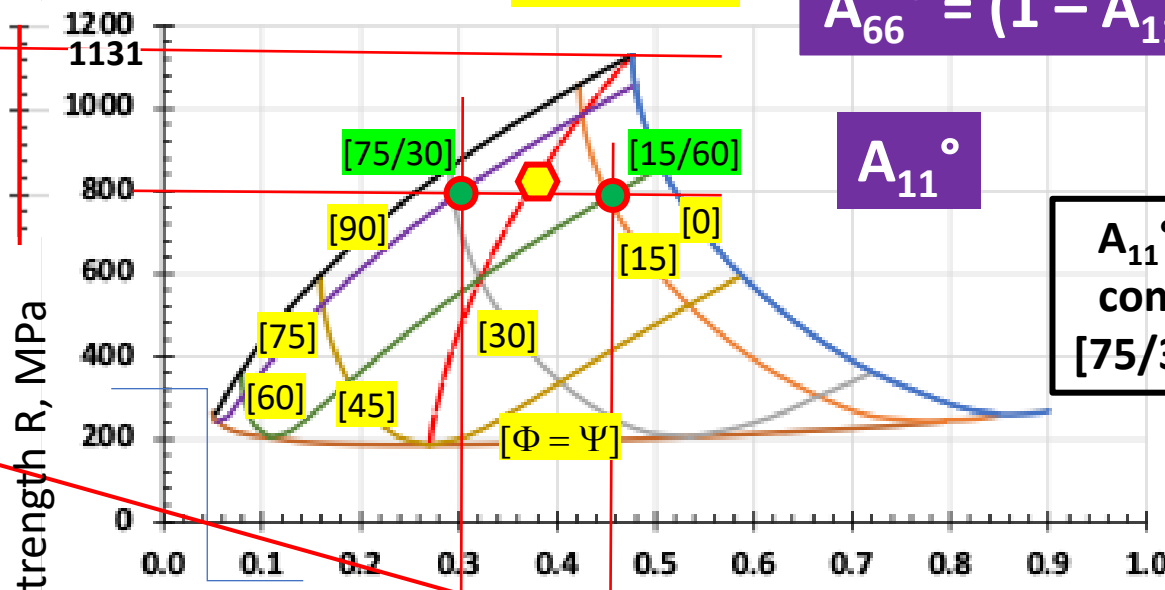
1520
1425
1260
1000

For all CFRP

$A_{66}° = (1 - A_{11}° - A_{22}°)/2$

$A_{11}°$

$A_{11}°$ and $A_{22}°$ are complementary: [75/30] for [15/60]

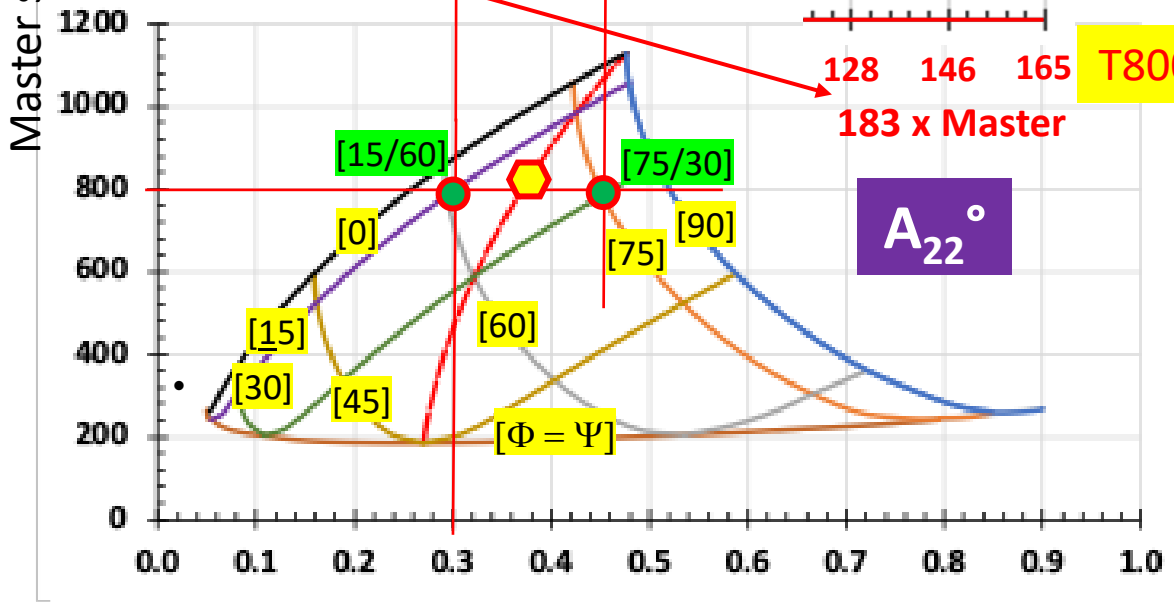


Master: A_{11}/trace

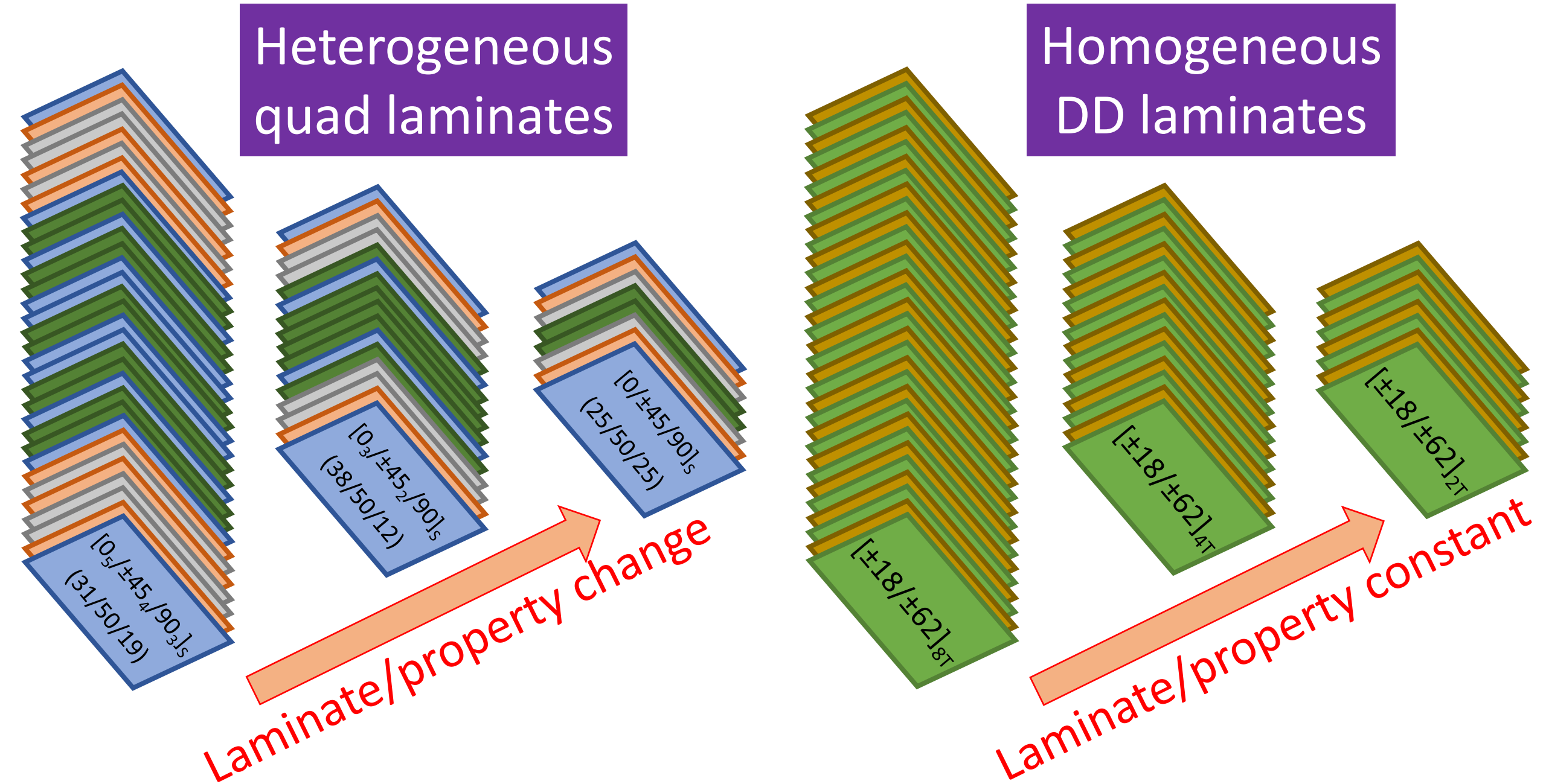
128 146 165 T800/Cyt, GPa

183 x Master

$A_{22}°$

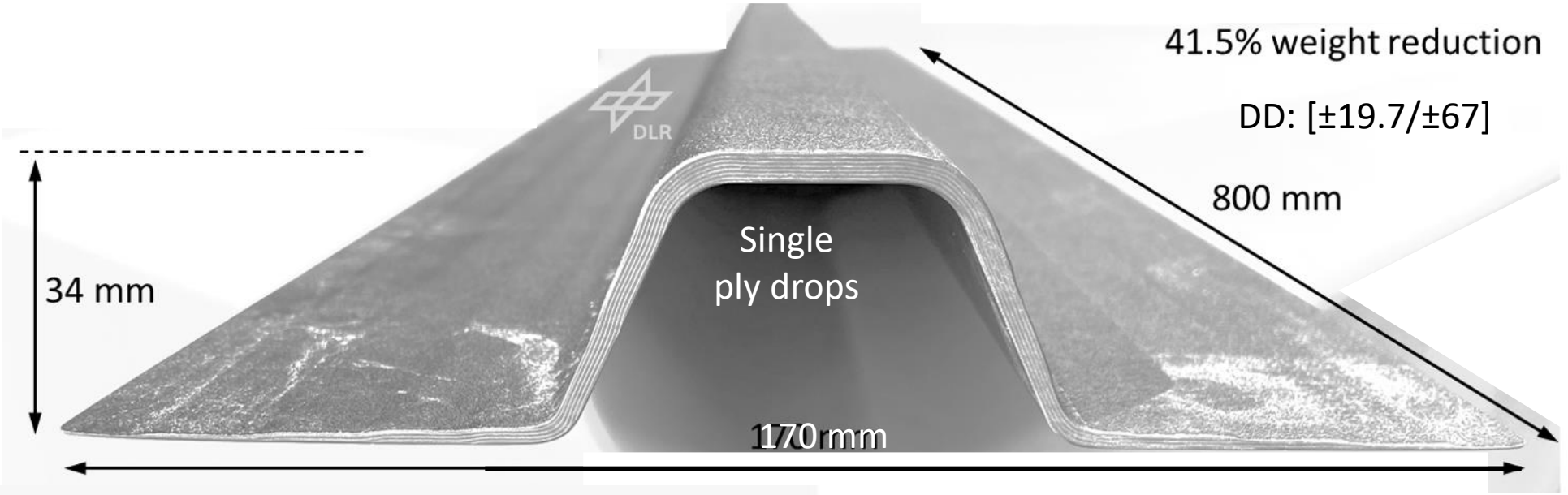


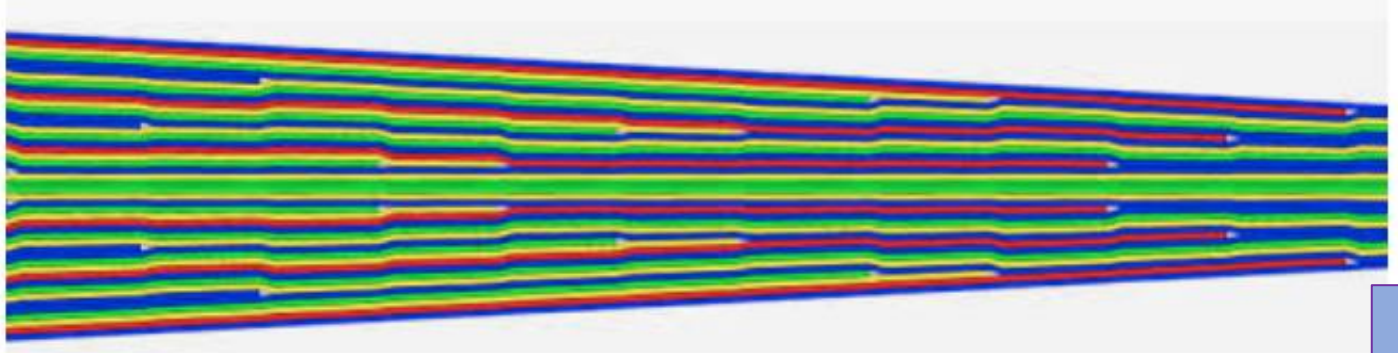
Master: A_{22}/trace


Heterogeneous quad laminates
[0₅/±45₄/90₃]s
(31/50/19)
[0₃/±45₂/90]s
(38/50/12)
[0/±45/90]s
(25/50/25)
Laminate/property change
Homogeneous DD laminates
[±18/±62]8T
[±18/±62]4T
[±18/±62]2T
Laminate/property constant

Weight reduction: 41.5% CFRP; <60% Alum

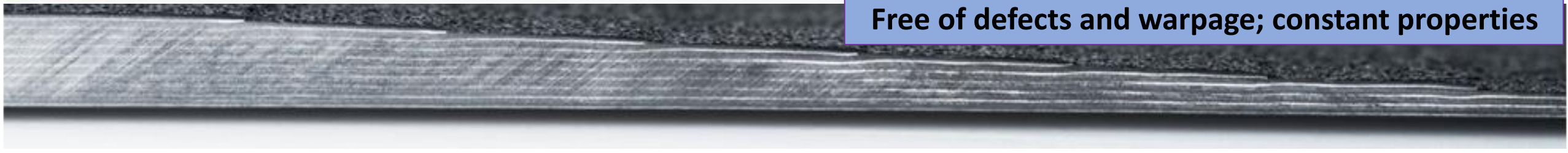
Erik Kappel





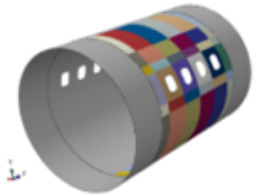
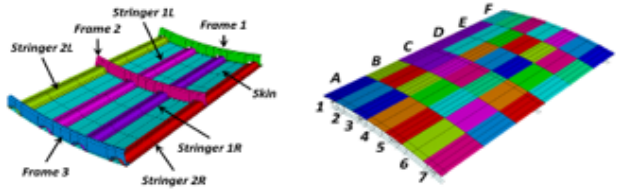
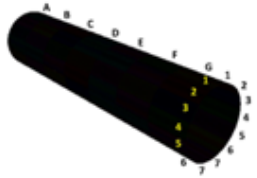
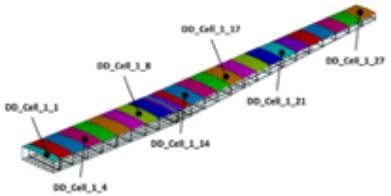
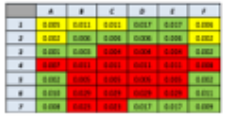
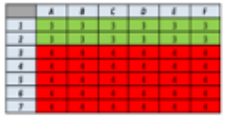
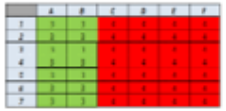
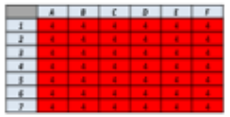
Interior ply drops in pairs: mid-plane symmetry;
Full of discontinuities; properties not constant

Double-double: exterior ply drops in singles:
Free of defects and warpage; constant properties

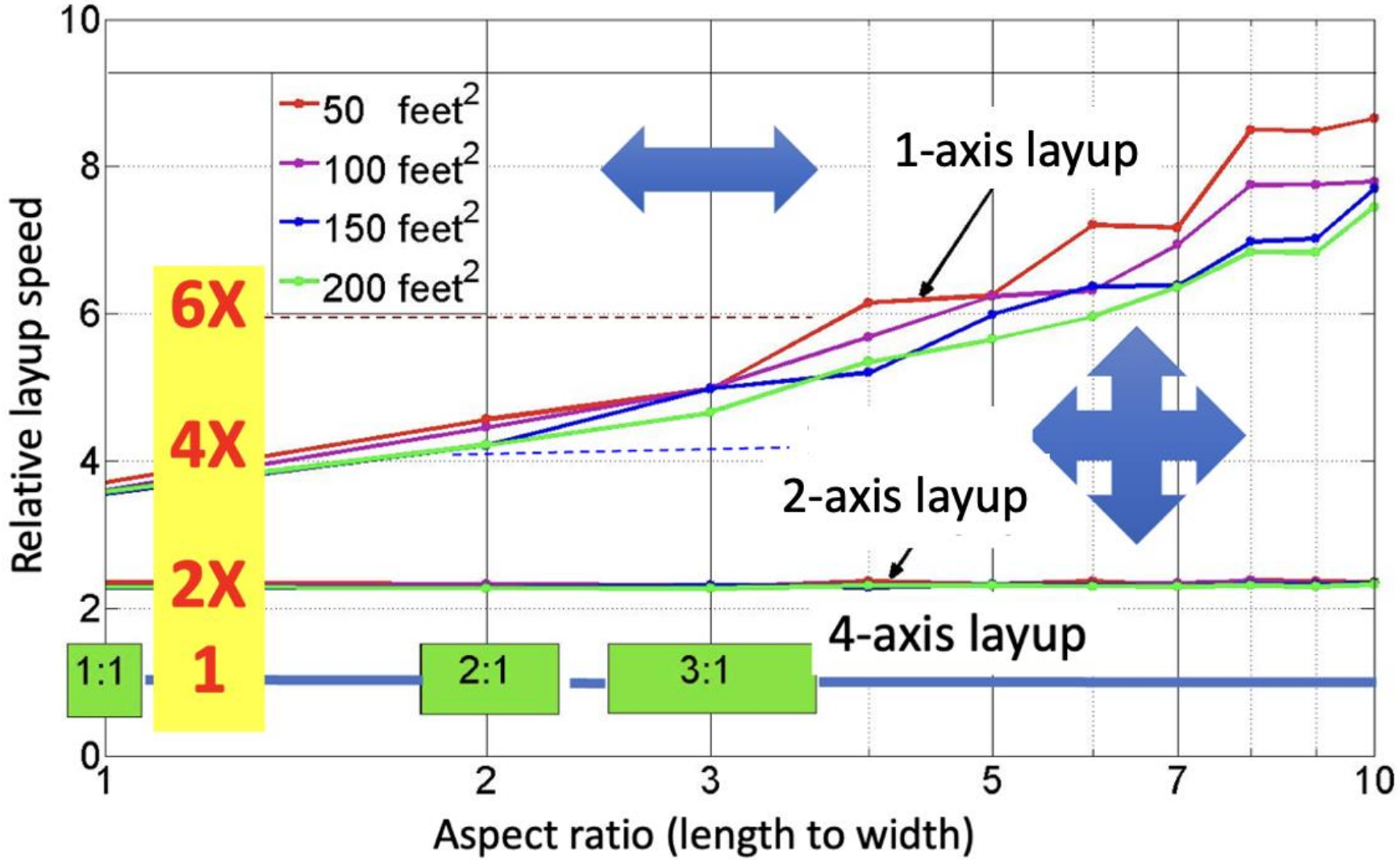


Weight savings from [QI] for various layups

Riccio & Di Carpio

Simpler layup	FUSELAGE  DD :Φ ±37.5; Ψ ±45	STIFFENED PANEL  DD :Φ ±15; Ψ ±15	TUBE  DD :Φ ±15; Ψ ±37.5	WING-BOX  DD: dep. on zone
DIGITIZED 	57 %	66 %	78 %	52 %
TAPERED 1 	52 %	63 %	58 %	≈25 %
TAPERED 2 	48 %	52 %	67 %	52 %
UNIFORM 	48 %	52 %	29 %	≈ 25 %

Tape deposition rates of 1-axis layup over 4-axis

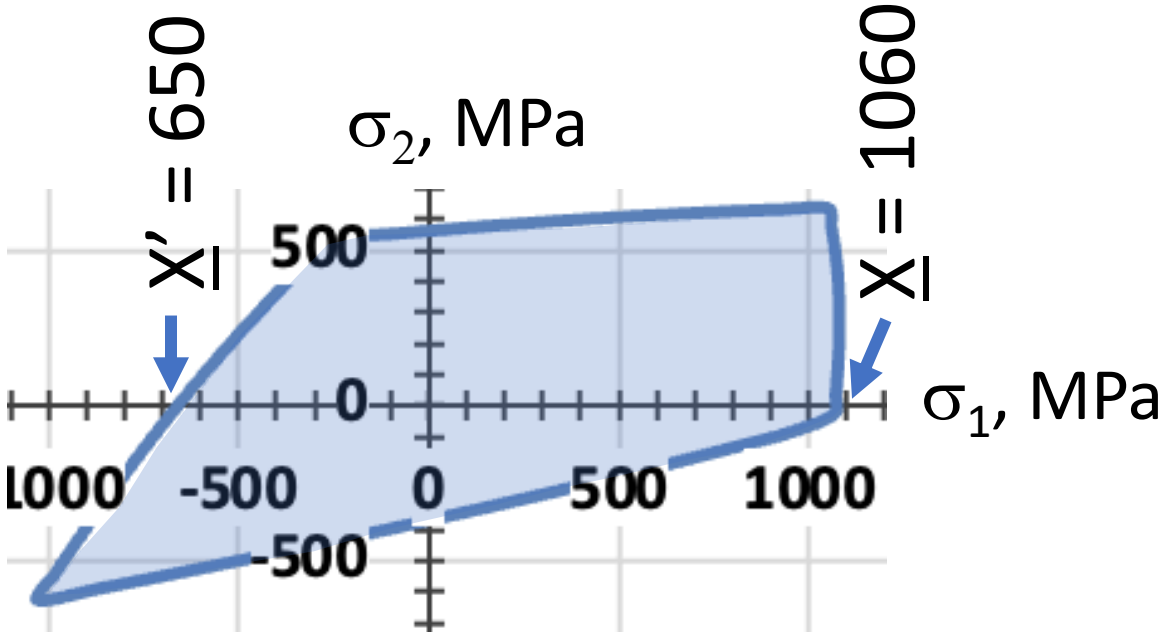


Pre-plied DD tape with thin plies: $[0/60/0/-60]_T$

Euro-Ace

T700/epoxy:
300 mm wide; 120 GSM

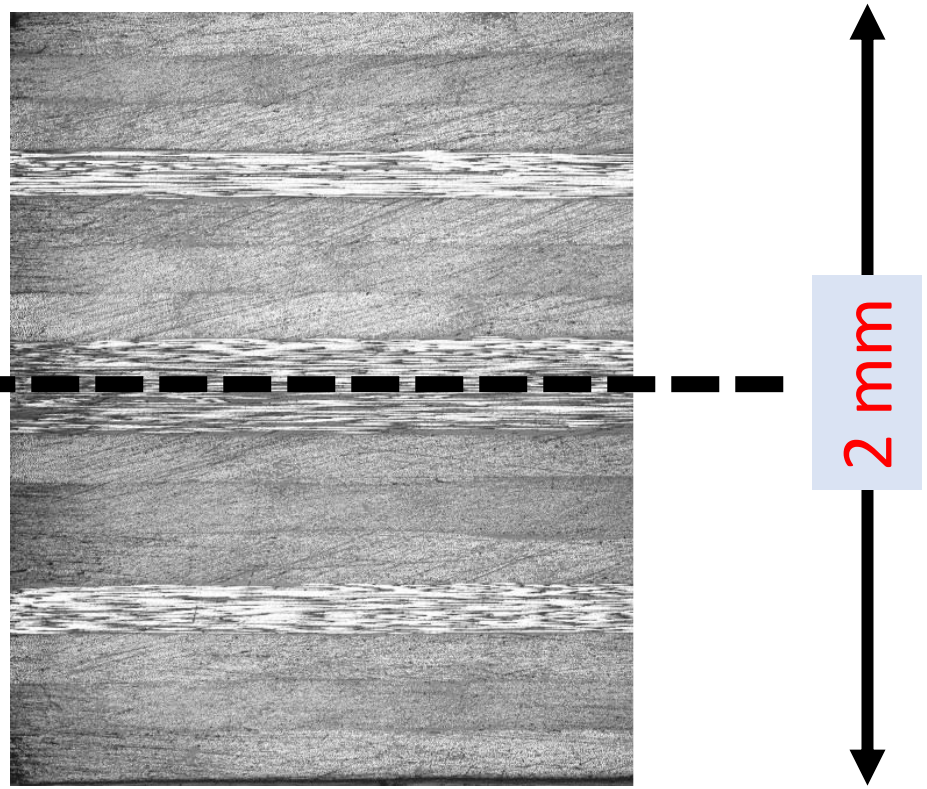
$E_1° = 60$, $E_2° = 35$, $E_6° = 13$ GPa

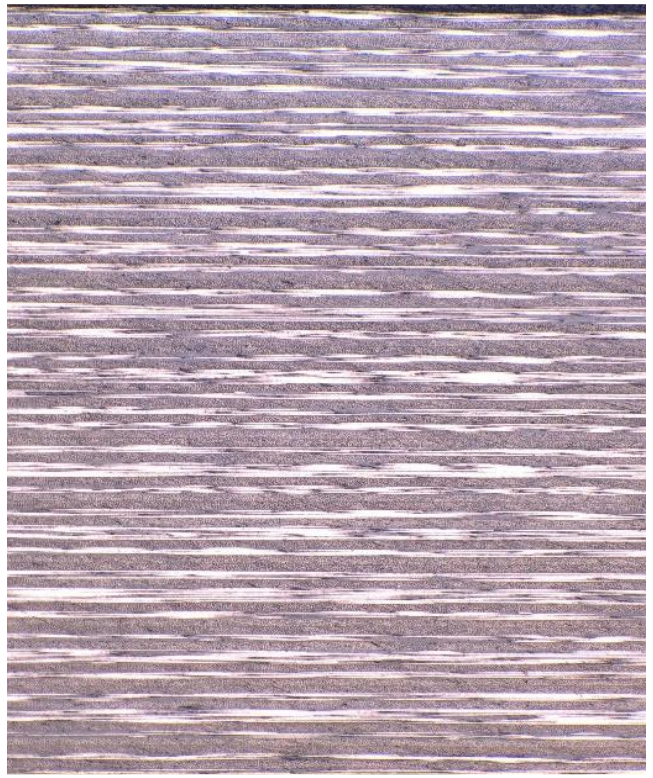


- Minimum gage < 1 mm
- 1-axis layup: 6X speed
- Homogenized < 1mm
- Easy tapering
- Ply drops in singles
- No interior defects

Quad: heterogeneous

PPDD: homogenized





$[+45/90/-45/0]_{2S}$

16 plies

$[+0/+60/0/-60]_{17T}$

17 layers

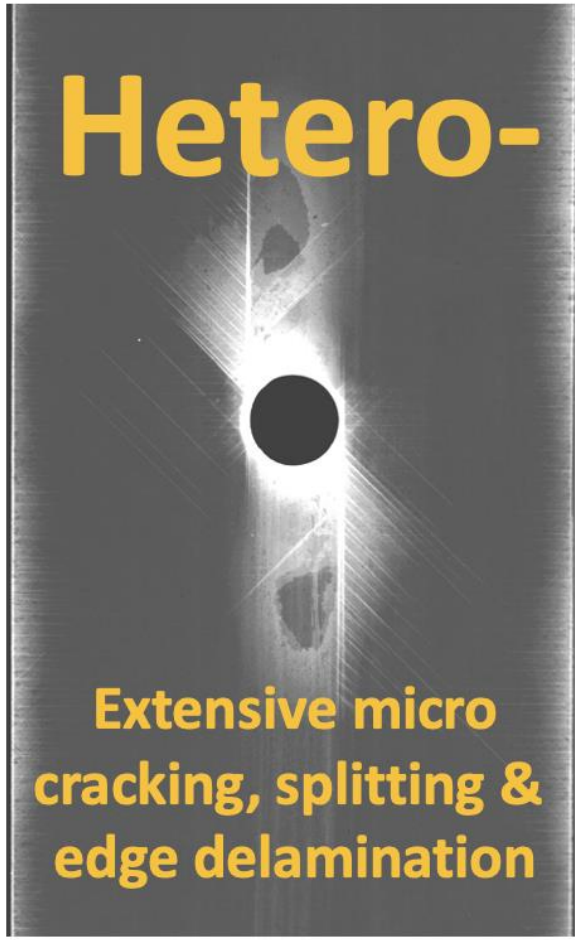
Hetero- vs homo-geneous hard laminate

Sangwook Sihn

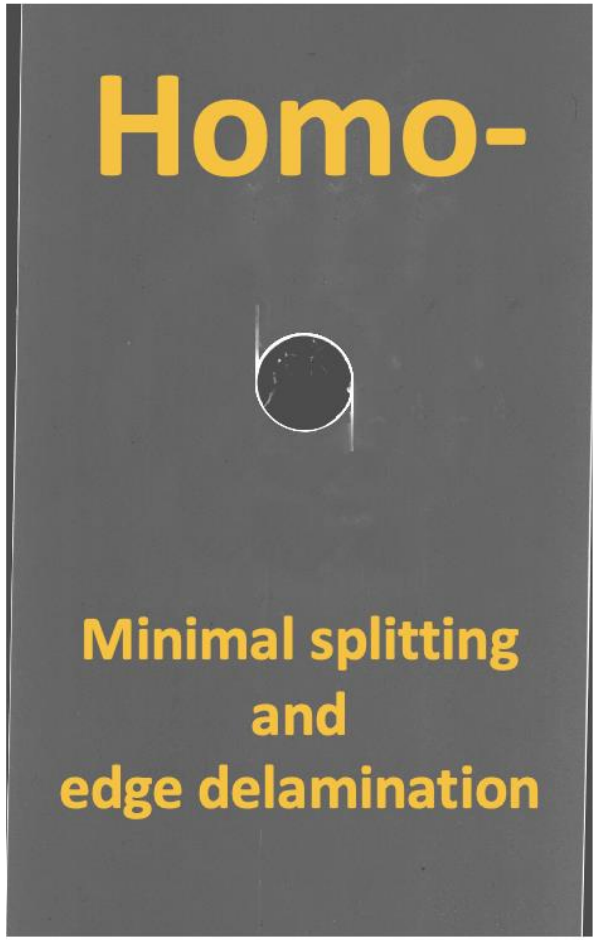
σ_{max} = 70 ksi (70% static), R = 0.1, f = 5 Hz, after 73,000 cycles

Ply thickness = 0.04 mm, Laminate thickness = 3.2 mm

$[45_5/0_{10}/-45_5/90_5/45_5/0_{10}/-45_5/0_5]_S$



$[45/0_2/-45/90/45/0_2/-45/0]_{5S}$



Opportunities

- DD is a straightforward application of laminated plate theory
- Simplified design, and testing
- Unique use of thin plies and pre-plying
- Significant weight and cost savings: 1-axis layup and tapering
- Need to learn scaling for generic solutions
- Need rapid testing for design allowable and certification
- Many new applications beyond aerospace and sporting goods

[1] S. W. Tsai, et al. DOUBLE-DOUBLE: a new perspective in the manufacture and design of composites, 2nd edition. Stanford, California: Stanford University, 2023
[2] S. W. Tsai, et al. DOUBLE-DOUBLE: a new perspective in the manufacture and design of composites. 1st edition. Stanford, California: Stanford University, 2022
[3] Tsai S. W., et al. Composite double-double and grid/skin structures Low Weight/Low Cost Design and Manufacturing, Stanford University (2019)
[4] S. G. Lekhnistkii. Anisotropic Plates, 2nd edition, English translation by S. W. Tsai and T. Cheron, Gordon-Breach. 1968

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[5] E. Kappel and S. W. Tsai. Double-Double laminates – a review on the status quo in the spring of 2024, March 2024. DOI
https://doi.org/10.57965/g2cb-8t21